Linearized stability analysis of accelerated planar and spherical fluid interfaces with slow compression

John D. Ramshaw and Peter A. Amendt
Lawrence Livermore National Laboratory, University of California, P.O. Box 808, L-097, Livermore, California 94551
(Received 5 November 2002; published 13 May 2003)

We present linearized stability analyses of the effect of slow anisotropic compression or expansion on the growth of perturbations at accelerated fluid interfaces in both planar and spherical geometries. The interface separates two fluids with different densities, compressibilities, and compression rates. We show that a perturbation of large mode number on a spherical interface grows at precisely the same rate as a similar perturbation on a planar interface subjected to the same normal and transverse compression rates.

DOI: 10.1103/PhysRevE.67.056304 PACS number(s): 47.20.Bp, 47.20.Ma

I. INTRODUCTION

There is an increasing interest in the effects of slow compression on the growth of small perturbations at unstable fluid interfaces [1,2]. The meaning of "slow" in this context is that the compression rates are assumed to be sufficiently small that the density remains essentially uniform in space within each fluid. This, in turn, requires the Mach number associated with the compression velocities to be small. However, a prolonged slow compression (or expansion) will eventually result in large density changes, so the resulting integrated compression need not, and, in general, will not, be small. Results of this type are also useful as simple approximations to the behavior at higher Mach numbers, as they capture the essential global effects of compression without the additional complications of a fully compressible treatment. Such results are useful in estimating the growth of interfacial instabilities during the implosion of inertial confinement fusion capsules [2], where compression effects are essential.

The effects of slow compression can readily be determined by the application of linear stability analysis to a slight generalization of the conventional potential flow theory [1,2]. Such analyses have recently been reported for accelerated shear layers in planar geometry [1] and for accelerated interfaces in spherical geometry [2]. In Ref. [2] the two fluids were allowed to have different compression rates, while Ref. [1] was restricted to the case in which the compression rates were the same in both fluids. Our main purpose here is to generalize the planar analysis of Ref. [1] to allow for unequal compression rates in the absence of shear. Thus we neglect the Kelvin-Helmholtz instability while retaining the Rayleigh-Taylor and Richtmyer-Meshkov instabilities and hybrid combinations thereof. The further generalization of the planar case to nonzero tangential velocities (i.e., accelerated shear layers) with different compression rates in the two fluids is considerably more complicated and will be presented elsewhere.

The present generalization of the planar analysis to unequal compression rates may then be compared to the corresponding spherical results of Amendt *et al.* [2]. For this purpose, however, it is convenient to slightly generalize the spherical analysis to allow the compression rate in the inner fluid to be independent of the interface motion. This makes the results more symmetrical and facilitates comparison with the planar case, where the compression rates are, of course, also independent of the acceleration of the interface. We also argue that this generalization of the spherical analysis is likely to provide a more accurate approximation in situations where the densities are no longer uniform within each fluid.

A comparison of the evolution equations for the perturbation amplitudes in the planar and spherical cases then shows that a perturbation with large Legendre mode number on a spherical interface grows at precisely the same rate as a similar perturbation on a planar interface with the same normal and transverse compression rates. This shows that interface curvature effects become negligible for large mode number, and has the practical consequence that in this limit the planar analysis is equally applicable in spherical geometry. A single formulation is therefore sufficient to describe and analyze both cases.

II. PLANAR GEOMETRY

In the absence of tangential velocities parallel to the interface, the planar case requires only straightforward modifications to the previous treatment of Ref. [1]. We consider an initially planar interface that separates two immiscible fluids with negligible surface tension in zero gravity. The unit normal to the interface is denoted by $\mathbf{n}$, which by convention points from fluid 1 into fluid 2. The system is subjected to a time-dependent normal acceleration $a(t)\mathbf{n}$ relative to an inertial laboratory frame. However, it is more convenient to describe the system in a comoving Cartesian coordinate frame in which the unperturbed interface remains stationary for all t. The interface is then defined by the time-independent equation $\mathbf{n}\cdot\mathbf{r}=0$, where $\mathbf{r}$ is the position vector relative to an origin located somewhere on the interface. The system then experiences an artificial external body force per unit mass of $-a(t)\mathbf{n}$ due to the acceleration of the coordinate frame.

Without yet perturbing the interface, we now suppose that the two fluids are being anisotropically compressed and/or expanded (i.e., strained) at rates that are uniform within each fluid but may differ between the two fluids. In the notation of Ref. [1], the unperturbed velocity of fluid i then becomes $\mathbf{u}_i^D=\mathsf{D}_i\cdot\mathbf{r}$, where the uniform symmetric tensor $\mathsf{D}_i=\nabla\mathbf{u}_i^D$ is the gradient of the velocity field associated with the compression or expansion within fluid i. Thus negative eigenvalues of D_i imply compression, while positive eigenvalues imply expansion in their respective principal directions. The normal components of the velocities $\mathbf{u}_i^D$ must, of course, be continuous at the interface, and indeed must vanish there for the interface to remain stationary. This requires $\mathbf{n}\cdot\mathsf{D}_i\cdot\mathbf{t}=0$, where $\mathbf{t}$ is any tangent vector normal to $\mathbf{n}$; i.e., $\mathbf{n}\cdot\mathbf{t}=0$. It follows that D_i must be of the form $\mathsf{D}_i=D_{ni}\mathbf{nn}+\mathsf{D}_{ti}$, where $\mathsf{D}_{ti}\cdot\mathbf{n}=0$, so that $D_{ni}=\mathbf{n}\cdot\mathsf{D}_i\cdot\mathbf{n}$. Moreover, in order to prevent the compression velocities from generating secondary Kelvin-Helmholtz instabilities, we must further require the tangential components of $\mathbf{u}_i^D$ to be continuous at the interface; i.e., $\mathbf{t}\cdot\mathsf{D}_{t1}\cdot\mathbf{t}=\mathbf{t}\cdot\mathsf{D}_{t2}\cdot\mathbf{t}$. Since $\mathbf{t}$ is an arbitrary tangential vector, this implies $\mathsf{D}_{t1}=\mathsf{D}_{t2}\equiv\mathsf{D}_t$. Thus only the normal components of D_1 and D_2 can differ, and D_i takes the form

$$\mathsf{D}_i=D_{ni}\mathbf{nn}+\mathsf{D}_t. \tag{1}$$

Since the compressions or expansions are uniform within each fluid, the fluid densities ρ_i also remain uniform within each fluid but now become dependent on time according to

$$\dot{\rho}_i=-D_i\rho_i, \tag{2}$$

where $D_i=\boldsymbol{\nabla}\cdot\mathbf{u}_i^D=\mathsf{D}_i:\mathsf{U}=D_{ni}+D_t$, $D_t=\mathsf{D}_t:\mathsf{U}$, and U is the unit dyadic.

The remainder of the analysis requires only straightforward modifications to the development of Ref. [1], so most of the details will be omitted. The interface is perturbed by subjecting each point $\mathbf{r}$ thereof to a small vectorial displacement $h(t)C\mathbf{n}$, where $C=\cos[\mathbf{k}(t)\cdot\mathbf{r}]$, $\mathbf{k}\cdot\mathbf{n}=0$, and $|h\mathbf{k}|\ll 1$ [3]. The interface is then defined by the equation $\mathbf{n}\cdot\mathbf{r}=hC$. As before, the time dependence of $\mathbf{k}$ is necessary to allow for the change in wavelength due to the tangential compression; i.e., nonzero D_t. This time dependence is determined by [1]

$$\dot{\mathbf{k}}=-\mathsf{D}_t\cdot\mathbf{k}=-\mathsf{D}\cdot\mathbf{k}, \tag{3}$$

which further implies that $\dot{k}=-D_k k$, where $k=|\mathbf{k}|$ and $k^2D_k=\mathbf{k}\cdot\mathsf{D}\cdot\mathbf{k}=\mathbf{k}\cdot\mathsf{D}_t\cdot\mathbf{k}$. The resulting potential flow field in fluid i to first order in h is given by $\mathbf{u}_i=\nabla\phi_i$, where $\phi_i=\phi_i^D+\phi_i'$, $\phi_i^D=\frac{1}{2}\mathbf{r}\cdot\mathsf{D}_i\cdot\mathbf{r}$, and

$$\phi_i'=\mp\frac{1}{k}(\dot{h}-D_{ni}h)Ce^{\mp k\mathbf{n}\cdot\mathbf{r}}, \tag{4}$$

where the upper sign applies for $\mathbf{n}\cdot\mathbf{r}>hC$ $(i=2)$, and the lower sign for $\mathbf{n}\cdot\mathbf{r}<hC$ $(i=1)$.

The pressures in the two fluids are determined by the generalized time-dependent Bernoulli equation [1]

$$p_i=-\rho_i\left(\frac{\partial\phi_i}{\partial t}+\tfrac{1}{2}|\nabla\phi_i|^2+a(t)\mathbf{n}\cdot\mathbf{r}\right)+p_i^0(t), \tag{5}$$

where $p_i^0(t)$ is a function of time alone. Setting $\mathbf{n}\cdot\mathbf{r}=hC$ and linearizing in h, we find

$$p_i=p_i^0(t)+p_i^C C, \tag{6}$$

where

$$kp_i^C=\pm\rho_i\left[\frac{d}{dt}(\dot{h}-D_{ni}h)+D_k(\dot{h}-D_{ni}h)\right]-\rho_i akh. \tag{7}$$

If Eq. (6) is spatially averaged over the interface, the cosine term averages to zero and drops out. Thus p_i^0 has the significance of the average pressure of fluid i, which is related to ρ_i by an appropriate equation of state: $p_i^0=f_i(\rho_i)$.

The dynamical evolution of the system is determined by requiring $p_1=p_2$ for all $\mathbf{k}\cdot\mathbf{r}$. The two terms in p_i in Eq. (6) are linearly independent, so their coefficients must be separately equal; i.e., $p_1^0(t)=p_2^0(t)$ and $p_1^C=p_2^C$. The former condition implies a relation between the overall compression rates in the two fluids, since it may be differentiated with respect to time to yield $\rho_1(df_1/d\rho_1)D_1=\rho_2(df_2/d\rho_2)D_2$. For isentropic compressions, this reduces to

$$\rho_1c_1^2D_1=\rho_2c_2^2D_2, \tag{8}$$

where c_i is the speed of sound in fluid i. Since $D_i=D_{ni}+D_t$, Eq. (8) is a constraint on the allowed values of D_{n1}, D_{n2}, and D_t, only two of which can be independently specified.

The condition $p_1^C=p_2^C$ determines the equation of motion for the linearized time evolution of the perturbation amplitude h. After some algebra, this equation reduces to

$$\sum_{i=1}^{2}\rho_i\left[\frac{d}{dt}(\dot{h}-D_{ni}h)+D_k(\dot{h}-D_{ni}h)\right]=\Delta\rho ka(t)h, \tag{9}$$

where $\Delta\rho=\rho_2-\rho_1$. In the special case when $\mathsf{D}_t=0$, so that $D_t=D_k=0$ and $D_{ni}=D_i$, Eq. (9) reduces to the infinite-thickness limit $(d_1,d_2\to\infty)$ of Eq. (62) of Goncharov *et al.* [4]. Since $\dot{k}=-D_k k$, Eq. (9) can be cast into the more compact form

$$\rho_1\frac{d}{dt}\left[\frac{1}{k}(\dot{h}-D_{n1}h)\right]+\rho_2\frac{d}{dt}\left[\frac{1}{k}(\dot{h}-D_{n2}h)\right]=\Delta\rho a(t)h. \tag{10}$$

Equation (9) can also be rewritten in an alternative form that more clearly exhibits the effects of unequal compression rates:

$$\frac{d}{dt}(\dot{h}-\overline{D}_nh)+D_k(\dot{h}-\overline{D}_nh)=k\frac{d}{dt}\left[\frac{1}{k}(\dot{h}-\overline{D}_nh)\right]=[Aka(t)+B(\Delta D)^2]h, \tag{11}$$

where $(\rho_1+\rho_2)\overline{D}_n=\rho_1D_{n1}+\rho_2D_{n2}$, $\Delta D=D_{n2}-D_{n1}=D_2-D_1$, $A=\Delta\rho/(\rho_2+\rho_1)$ is the Atwood number and $B=\rho_1\rho_2/(\rho_1+\rho_2)^2$. In the special case where $D_{n1}=D_{n2}\equiv D_n$, $\overline{D}_n$ reduces to D_n, $\Delta D=0$, and Eq. (11) reduces to the result of setting the transverse velocity difference $\boldsymbol{\Delta}\mathbf{u}=0$ in Eq. (22) of Ref. [1]. In the general case where D_{n1}

$\neq D_{n2}$, we see that the time derivative of the net instability growth rate $\dot{h}-\overline{D}_n h$ is increased by the term $B(\Delta D)^2 h$, in which the coefficient of h is positive definite. It is interesting to note that this term enters into the evolution equation in much the same way as the Kelvin-Helmholtz growth term $B(\mathbf{k}\cdot\Delta\mathbf{u})^2 h$ in Eq. (22) of Ref. [1]. It even has the same density dependence, but it has a different wavelength dependence, being of zeroth rather than second order in k.

It is also of interest to reexpress Eq. (9) in terms of the masses transported across the original unperturbed interface by the instability, which removes the purely geometrical effects of normal compression on the perturbation amplitude h [1]. The mass of fluid i that has moved across some Lagrangian area $\mathcal{A}$ of the original interface by time t is given by $M_i(t)=\pi^{-1}\rho_i\mathcal{A}h$, where $\dot{\mathcal{A}}=D_t\mathcal{A}$ due to the transverse compression [1]. The total mass having crossed that area by time t is therefore $M=M_1+M_2$, the evolution equation for which is readily found to be

$$\ddot{M}+(\overline{D}_n+D_k)\dot{M}=[Aka(t)+B(\Delta D)^2]M, \qquad (12)$$

which generalizes Eq. (23) of Ref. [1] to nonzero ΔD. Equation (12) shows that when the purely geometrical effects of normal compression on h are removed, the remaining dynamical effects of compression enhance the instability growth in two ways: (a) the mean compression rate increases $\ddot{M}$ by a term proportional to $\dot{M}$, just as before [1], while (b) the difference between compression rates further increases $\ddot{M}$ by a term proportional to M.

III. SPHERICAL GEOMETRY

An analogous analysis of the effects of slow compression on instability growth at accelerated spherical interfaces has recently been presented by Amendt *et al.* [2]. It is of interest to compare the planar and spherical cases to obtain insight into how and why they differ, and the circumstances under which one may or may not be accurately approximated by the other. For this purpose, however, it is convenient to slightly generalize the analysis of Ref. [2] as described below.

We consider an initially spherical interface $r=R(t)$ in spherical polar coordinates, where r is the radial coordinate. This interface separates an inner fluid 1 and an outer fluid 2 with different densities and compressibilities. The unit normal $\mathbf{n}$ to the unperturbed interface is then identical to the unit vector in the radial direction. The unperturbed velocity field $\mathbf{u}_i^D$ in fluid i is assumed to be purely radial with uniform divergence D_i. Thus $\mathbf{u}_i^D=u_i\mathbf{n}$ and $r^2D_i=(\partial/\partial r)(r^2u_i)$, so that $u_i=\frac{1}{3}D_ir+E_i/r^2$, where E_i is an integration constant. Since D_i is uniform within each fluid, the fluid densities continue to obey Eq. (2). The unperturbed interface is a Lagrangian surface, so the velocities u_i must be continuous with the common value $\dot{R}$ at $r=R$. This implies

$$E_i=R^2\dot{R}-\tfrac{1}{3}R^3D_i, \qquad (13)$$

which determines E_i in terms of D_i and the motion of the interface. Combining Eqs. (2) and (13) for $i=1$, we find $4\pi\rho_1E_1=\dot{m}_1$, where $m_1=(4\pi/3)R^3\rho_1$ is the total mass of the inner fluid 1. Thus a nonzero value of E_1 implies a velocity singularity and a corresponding mass source or sink of strength $4\pi\rho_1E_1$ at the origin. In Ref. [2], this singularity and mass source were ruled out on physical grounds and were therefore removed by setting $E_1=0$, which implies $\dot{m}_1=0$ and $D_1=3\dot{R}/R$. Here, however, we shall leave E_1 arbitrary, thereby allowing D_1 to be specified independently of the interface motion. This leads to more symmetrical results and facilitates the comparison to the planar case, where D_1 is, of course, also independent of the acceleration of the interface.

The velocity gradient tensor $\mathsf{D}_i=\nabla\mathbf{u}_i^D$ in fluid i is given by

$$\mathsf{D}_i=\frac{\partial u_i}{\partial r}\mathbf{nn}+\frac{u_i}{r}(\mathsf{U}-\mathbf{nn})=\left(\frac{1}{3}D_i-\frac{2E_i}{r^3}\right)\mathbf{nn}+\left(\frac{1}{3}D_i+\frac{E_i}{r^3}\right)(\mathsf{U}-\mathbf{nn}). \qquad (14)$$

In contrast to the planar case, we see that D_i is no longer spatially uniform within each fluid, even though its trace D_i is uniform by assumption.

It is straightforward to repeat the analysis of Ref. [2] for an arbitrary value of E_1, so the details will again be omitted. The final result may be written as

$$\frac{\rho_1}{\ell}\frac{d}{dt}[R(\dot{h}-D_{n1}h)]+\frac{\rho_2}{\ell+1}\frac{d}{dt}[R(\dot{h}-D_{n2}h)]=\Delta\rho\ddot{R}h, \qquad (15)$$

where ℓ is the Legendre mode number of the perturbation [2], and $D_{ni}=D_i-2\dot{R}/R$, which is simply the value of $\mathbf{n}\cdot\mathsf{D}_i\cdot\mathbf{n}=(\partial u_i/\partial r)$ at $r=R$. Equation (15) may be rewritten in the equivalent form

$$\frac{\rho_1}{\ell}\frac{d}{dt}\left[\frac{1}{\rho_1R}\frac{d}{dt}(\rho_1R^2h)\right]+\frac{\rho_2}{\ell+1}\frac{d}{dt}\left[\frac{1}{\rho_2R}\frac{d}{dt}(\rho_2R^2h)\right]=\Delta\rho\ddot{R}h. \qquad (16)$$

Equation (16) is identical to the infinite-shell-thickness limit ($r_a\to\infty$) of Eq. (209) of Goncharov *et al.* [4], which they derived under the assumption that $E_1=0$. The present analysis shows that Eq. (15) or (16) is more generally valid for an arbitrary value of E_1.

When the derivatives are expanded, Eq. (15) takes the form

$$\ddot{h}+\left(\frac{3\dot{R}}{R}-V_\ell\right)\dot{h}-\left(\frac{U_\ell\ddot{R}}{R}+\frac{V_\ell\dot{R}}{R}+W_\ell\right)h=0, \qquad (17)$$

where

$$U_\ell = \frac{\ell(\ell-1)\rho_2 - (\ell+1)(\ell+2)\rho_1}{(\ell+1)\rho_1 + \ell\rho_2}, \tag{18}$$

$$V_\ell = \frac{(\ell+1)\rho_1 D_1 + \ell\rho_2 D_2}{(\ell+1)\rho_1 + \ell\rho_2}, \tag{19}$$

$$W_\ell = \frac{(\ell+1)\rho_1 \dot{D}_1 + \ell\rho_2 \dot{D}_2}{(\ell+1)\rho_1 + \ell\rho_2}. \tag{20}$$

An expression of the same form as Eq. (17) was derived some years ago in a widely circulated unpublished memorandum by Fisher [5], but with an unfortunate algebraic error in the term $V_\ell \dot{R}h/R$ [2]. In the special case where $E_1=0$, D_1 reduces to $3\dot{R}/R$, and Eq. (17) then reduces, after some algebra, to Eq. (4b) of Ref. [2]. If we further specialize to situations in which the outer fluid is incompressible ($D_2 = 0$), we recover a recent result of Lin *et al.* [6].

It is instructive to compare the planar case with the spherical case in the limit of large mode number, $\ell \gg 1$. In this limit the effective wave number of the perturbation becomes $k = \ell/R$ [1], and Eq. (15) reduces immediately to Eq. (10) with the planar acceleration $a(t)$ replaced by the spherical acceleration $\ddot{R}$. The planar and spherical cases with compression therefore become identical in the limit of large mode number, just as they do for incompressible fluids [1]. This shows that there are no residual curvature effects in this limit, and that the remaining effects of the spherical geometry are entirely due to the transverse compression resulting from the convergence of radial lines to the origin. This convergence effect is unrelated to curvature and arises because the transverse compression rate D_t cannot be specified independently in spherical coordinates but is inherently determined geometrically by the interface motion via the identity $D_t = D_i - D_{ni} = 2\dot{R}/R$. It follows that curvature effects intrinsic to spherical geometry are significant only for low mode numbers, and all remaining spherical convergence effects are merely transverse compression effects that are correctly captured by the planar stability analysis, provided that k and D_t are replaced by their appropriate spherical values as given above. This provides a useful economy of description, since both the planar case and the spherical case for $\ell \gg 1$ can now be analyzed using the planar results of Sec. II.

Finally, we remark that Eq. (15), (16), or (17) may also be useful as an approximation when the fluid densities ρ_i are not strictly uniform within each fluid as assumed in the analysis. In such situations, one would intuitively expect these equations to provide a good approximation provided that (a) the fluid densities remain essentially uniform over distances of order h [7] and (b) ρ_i, D_{ni}, and D_i are interpreted as their *local* values immediately adjacent to the interface. Since Eq. (2) applies locally in a Lagrangian sense, it may be used to determine the local interfacial values of D_i from the local values of ρ_i and $\dot{\rho}_i$. This, in turn, determines the local interfacial values of $D_{ni} = D_i - 2\dot{R}/R$. If ρ_1 is nonuniform within fluid 1, then D_1 must, of course, be nonuniform as well, and its local value near the interface will in general no longer be $3\dot{R}/R$. A nonzero value of E_1 is then required to mimic this situation in the present analysis. The corresponding mass source or sink at the origin is artificial but harmless; it merely represents the mass source or sink that would be required if the true local values of ρ_1 and D_1 near the interface were to be maintained uniformly everywhere within fluid 1.

IV. SUMMARY

Linear stability analyses have been performed to determine the effect of slow anisotropic compression or expansion on the growth of perturbations at accelerated planar and spherical interfaces between two fluids with different densities, compressibilities, and compression rates. The resulting ordinary differential equations that govern the time dependence of the perturbation amplitudes are given by Eq. (9), (10) or (11) in the planar case, and by Eq. (15), (16), or (17) in the spherical case. We have also shown that when the transverse compression rates in the planar case are properly specialized to those in the spherical case, the perturbation growth rates in the two cases become identical for large mode number. This correspondence implies that interface curvature effects become negligible in that limit, and that the remaining "convergence" effects commonly associated with spherical geometry are in reality merely transverse compression effects that are completely and correctly captured by a planar stability analysis. This observation lends some valuable insight into the intrinsic nature of curvature, compression, and convergence effects, and helps to clarify the distinctions between them. It also has the practical advantage that the same equations can now be used to analyze both cases.

ACKNOWLEDGMENTS

We are grateful to Karnig Mikaelian for many helpful discussions and to Bill Moran for thoughtful comments on the manuscript. This work was performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48.

[1] J.D. Ramshaw, Phys. Rev. E **61**, 1486 (2000); **64**, 069904(E) (2001).

[2] P. Amendt, J.D. Colvin, J.D. Ramshaw, H.F. Robey, and O.L. Landen, Phys. Plasmas **10**, 820 (2003).

[3] In practice, it is found that nonlinear effects become significant and the linear approximation becomes invalid when $|h\mathbf{k}| \gtrsim 1/2$; see, for example, S.W. Haan, Phys. Rev. A **39**, 5812 (1989), and references cited therein. The time at which this occurs cannot, in general, be estimated in advance, as it depends strongly on the acceleration history $a(t)$ as well as the initial perturbation amplitude h_0 and the compression rates. For a constant acceleration a_0 in the absence of compression,

this time is given by $t \cong (kAa_0)^{-1/2}\cosh^{-1}(1/[2|h_0|k])$, where $|h_0|k < 1/2$, $Aa_0 > 0$, and A is the Atwood number.
[4] V.N. Goncharov, P. McKenty, S. Skupsky, R. Betti, R.L. McCrory, and C. Cherfils-Clérouin, Phys. Plasmas **7**, 5118 (2000).
[5] H.N. Fisher, Los Alamos National Laboratory internal memorandum X-1(5/82)22, September 1982 (unpublished).
[6] H. Lin, B.D. Storey, and A.J. Szeri, Phys. Fluids **14**, 2925 (2002).
[7] The essential physical condition for this uniformity is that the time required for sound waves to travel a distance of order $|h|$ in fluid i be much shorter than the time scale $|D_i|^{-1}$ for the density to change significantly; i.e., $|D_i h| \ll c_i$. As mentioned in the Introduction, this condition is equivalent to a low Mach number restriction, since $|D_i h|$ is the change in compression velocity over a distance of order $|h|$.